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Modeling and control of micro-mechatronic devices: application of variational and energetic methods for micro-actuator design

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***Abstract*— This paper is focused on a modeling procedure well-suited for the design of micro-mechatronic systems and especially for micro-actuators. The purpose of this publication is to show that the variational and energetical methods is not only well-suited to model *classical* micro-mechatronic devices but that they are also well-suited to include complex dynamical behaviour such as non-linearity and hysteretical behaviour. This procedure is applied to the design of a new actuator using one of the relatively new smart materials, the Magnetic Shape Memory Alloys (MSMAs). It should be stressed that the presented approach can be extented to a great range of other smart materials and that the description can be easily extented up to the control level.**

I. Introduction

Micro-mechatronic devices are systems including mechatronic elements at the mini- and microscopic scale. Presently, most of these devices concern actuators and sensors applications. Indeed, at this scale, the use of multi-components systems to develop a mechatronic function is not convenient because of integration difficulties. Therefore a great part of these applications use smart materials because they can achieve a sensing and actuation function in an integrated and distributed way.

This paper is focused on a modeling procedure well-suited to micro-actuators design. In a first step, some headlines will be given concerning variational and energetic methods for the modeling and design procedures. These procedures will be applied for the design of a new actuator using one of the relatively new smart materials, the Magnetic Shape Memory Alloys (MSMAs). Even if, this method is applied to MSMAs materials, it should be stressed that this approach can be extented to a great range of other smart materials.

II. Variational and energetic methods for the micro-actuator design

For the modeling of any micro-mechatronic systems, the design has to take into account different physical fields. For the example of an MSMA actuator, a *thermo-magneto-mechanical* model is required. During the first design step of *classical* actuators (not requiring a mechatronic approach), each physical field is usually studied independently and the coupling effects are taken into account in a second step using numerical computations such as the Finite Elements Analysis (FEA). Nevertheless, for mechatronic systems and especially when the size of the device is reduced, the coupling effects become predominant as it is the case for the use of smart materials. The coupling effects must then be taken into account from the first step of the design procedure. At this stage, the FEA are not usually relevant because of the computing time and the necessity to already have the geometry of the devices at one's disposal. Indeed, it is necessary to use a *synthesis* tool for the design procedure instead of an *analysis* tool to make the first design choices. This tool must also be multi-physical and in this purpose, the only relevant method is to adopt an energetic point of view because energy is the only common point between different fields of physics.

For the mechatronic field, such a relevant tool is the *Bond Graph* method. It is a muti-physical, graphical and energetical tool well-suited for the design procedure. The last edition of the reference book on the subject [1] emphasises this point by adding the subtitle *Modeling and Simulation of Mechatronic Systems* to the title of the book. As the Bond-graph method is a relevant and efficient tool for mechatronic devices design and modeling, it is nevertheless limited for micro-mechatronic devices because it is difficult to model *distributed parameters systems* described by *Partial Differential Equations* with this tool. In such a case, variational methods are much more relevant. These variational methods in an energetical context include the Lagrangian and Hamiltonian formalisms ([2], [3], [4], [5]). These methods can model discret parameters systems (lumped parameters physics) as well as distributed parameters systems (continuum physics) in a similar way. These methods are also the bases of approximate methods such as the Finite Elements Analysis Technique. Some recent textbooks on mechatronic modeling emphasises this point in beeing completely based on the Lagrangian modeling for mechatronic devices as well as for micro-mechatronic devices using piezoelectric materials ([6], [7]).

A. Conservative systems

The lagrangian formalism is a modeling technique based on some energy functions used with the Hamilton principle [4] [5]. This principle postulates that the variation of an action $\mathcal{S}$ between two times on a real path is always equal to zero. This action is the lagrangian $\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}, t)$ integrated between the two

times t_1 et t_2 :

$$\mathcal{S} = \int_{t_1}^{t_2} \mathcal{L}\, dt \;\Rightarrow\; \delta S = 0 \text{ on a real path} \tag{1}$$

For the particular case of non-relativist systems, the lagrangian function is the difference between a kinetic co-energy $\mathcal{T}^*(\dot{\mathbf{q}})$ and a potential energy $\mathcal{V}(\mathbf{q})$ [2]. For conservative systems (i.e. closed and non-dissipative), $\mathcal{L}$ does not depend explicitely on the time t:

$$\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) = \mathcal{T}^*(\dot{\mathbf{q}}) - \mathcal{V}(\mathbf{q}) \tag{2}$$

A variational calculus on the Hamilton principle leads to the set of n Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) = 0 \qquad i \in [1, n] \tag{3}$$

where $\frac{\partial \mathcal{L}}{\partial q_i}$ are the *generalized forces* and $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ are the *generalized momentums*.

The Hamilton formalism is an extension of the lagrangian formalism which uses a Legendre transformation to substitute the time rate functions $\dot{\mathbf{q}}$ in the lagrangian $\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}})$ for the generalized momentum $\mathbf{p} = \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}$ in a new energy function called the *hamiltonian* function $\mathcal{H}(\mathbf{q}, \mathbf{p})$:

$$\mathcal{H}(\mathbf{q}, \mathbf{p}) = \mathbf{p} \cdot \dot{\mathbf{q}} - \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) \tag{4}$$

In such a case, the hamiltonian function corresponds to the total energy expressed with coordinates $\mathbf{q}$ and momentums $\mathbf{p}$ instead of coordinates $\mathbf{q}$ and velocities $\dot{\mathbf{q}}$:

$$\mathcal{H}(\mathbf{q}, \mathbf{p}) = \mathcal{T}(\mathbf{p}) + \mathcal{V}(\mathbf{q}) \tag{5}$$

Then the n second order Lagrange equations are transformed into a set of $2n$ first order Hamilton equations:

$$\begin{cases} \dot{q}_i = \dfrac{\partial \mathcal{H}}{\partial p_i} \\ \dot{p}_i = -\dfrac{\partial \mathcal{H}}{\partial q_i} \end{cases} \qquad i \in [1, n] \tag{6}$$

With this formalism, $\mathbf{x} = \begin{pmatrix} \mathbf{q} & \mathbf{p} \end{pmatrix}^T$ can be chosen as the state of the system and (6) becomes the state-space model of the system:

$$\frac{d}{dt}\begin{pmatrix} \mathbf{q} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{f}_{\mathbf{q}}(\mathbf{q}, \mathbf{p}) \\ \mathbf{f}_{\mathbf{p}}(\mathbf{q}, \mathbf{p}) \end{pmatrix} \tag{7}$$

B. Dissipative and controlled systems

The previous variational procedures are defined for conservative systems, nevertheless this formalism can be extended to the non-conservative case (*open* and *dissipative*) and also to systems that include *kinematic constraints*.

An *open* system means that it exchanges some energy with an outer system. This is the case when we try to control the system with an external force $\mathbf{f}_{ext}$ depending explicitely on the time t. In this case, we also speak about *non-autonomous* systems. A *dissipative* system means that some part of the inner energy is non-available during *any motion* of the system. A system with *kinematic constraints* mean that there is some geometric or topologic constraints on the admissible motion of this system. These kinematic constraints can be taken into account in the dynamical equations using a Lagrange multiplier technique.

To take into account all these phenomena, we make use of an *extented lagrangian function* $\mathcal{L}'$:

- the external generalized forces $\mathbf{f}_{ext}(\mathbf{q}, t)$ are taking into account in the variation of $\mathcal{L}'$ by adding their virtual works $\delta\mathcal{W}_{ext} = \mathbf{f}_{ext}(\mathbf{q}, t) \cdot \delta\mathbf{q}$;
- dissipations by static and viscous frictions are taking into account by adding their dissipated energies variations $\delta\mathcal{Q}_s(\mathbf{q})$ and $\delta\mathcal{Q}_v(\dot{\mathbf{q}})$. The dissipation by viscous friction $\mathcal{Q}_v(\dot{\mathbf{q}})$ is calculated with a Rayleigh dissipation function $\mathcal{R}(\dot{\mathbf{q}})$ like $\mathcal{Q}_v(\dot{\mathbf{q}}) = \int_{t_1}^{t_2} \mathcal{R}(\dot{\mathbf{q}})\, dt$;
- the *holonomic* kinematic constraints $\mathbf{c}(\mathbf{q}) = 0$ are taking into account with a Lagrange multipliers technique by adding the term $-\lambda \cdot \delta\mathbf{c}(\mathbf{q})$ to the variation $\delta\mathcal{L}'$.

Finally, we have:

$$\delta\mathcal{L}' = \delta\mathcal{L} + \mathbf{f}_{ext} \cdot \delta\mathbf{q} + \delta\mathcal{Q}_s + \delta\mathcal{Q}_v - \lambda \cdot \delta\mathbf{c} \tag{8}$$

The Hamilton principle using $\delta\mathcal{L}'$ gives the following Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{R}}{\partial \dot{q}_i} + \frac{\partial \mathcal{Q}_s}{\partial q_i} + f_{ext,i} - \lambda_i \cdot \frac{\partial c_i}{\partial q_i} = 0 \tag{9}$$

The extented lagrangian function for a controlled dissipative system with kinematic constraints can also be transformed into an extented hamiltonian function. This leads to the following Hamilton equations:

$$\begin{cases} \dot{q}_i = \dfrac{\partial \mathcal{H}}{\partial p_i} \\ \dot{p}_i = -\dfrac{\partial \mathcal{H}}{\partial q_i} - \dfrac{\partial \mathcal{R}}{\partial \dot{q}_i} + \dfrac{\partial \mathcal{Q}_s}{\partial q_i} + f_{ext,i} - \lambda_i \cdot \dfrac{\partial c_i}{\partial q_i} \end{cases} \tag{10}$$

C. Thermodynamics of irreversible processes as a way to define dissipative potential for nonlinear systems

The previous Lagrange and Hamilton formalisms are well-suited to conservative systems and can also be extented to take into account the dissipative and exchanges phenomona. The connexion with thermodynamics is therefore direct because, except for the choice of the relevant coordinates, the hamiltonian function $\mathcal{H}(\mathbf{q}, \mathbf{p})$ corresponds to the total energy $\mathcal{E}_T(\mathbf{q}, \dot{\mathbf{q}})$, the energetical function used in thermodynamics. The point of view of thermodynamics of irreversible processes is a phenomenological point of view directed toward the storage, the available and the dissipated energies.

The first law of the thermodynamic states that the total energy – or the hamiltonian – is always conserved in a close system and its variation gives rise to an energy exchange in an open system [8]. The exchange appears through an heat exchange $\mathcal{Q}_e$ and a work exchange $\mathcal{W}_e$ corresponding in particular to mechanical, electromechanical, thermal, chemical,..., exchange processes. The energy exchange depends on the path and then, work and heat exchanges are not state variables:

$$d\mathcal{E}_T = đ\mathcal{Q}_e + đ\mathcal{W}_e \tag{11}$$

In thermodynamics, the total energy is not the only relevant energetical functional and some other thermodynamic state functions as the internal energy $\mathcal{U}$, the Helmholtz Free energy $\mathcal{F}$, the Gibbs free energy $\mathcal{G}$ can be used instead. They are defined using a Legendre transformation applied to the total energy as it was the case between the lagrangian and the hamiltonian functions.

In thermodynamical systems, we must notice a strong distinction between the work and heat exchange because when the first one is reversible, the second one is always irreversible. It implies that some energy is unavaible in the stored energy. The *lost* energy contributes to the increase of the total entropy ($đS_i \geq 0$ with S_i the irreversible entropy). This is stated as the second law of the thermodynamic and leads to the Clausius-Duhem inequality of irreversible behaviour. With T the temperature of the system and $đ\mathcal{D}$ the increment of dissipation, we have:

$$T \cdot đS_i = đ\mathcal{D} \geq 0 \tag{12}$$

The purpose of thermodynamics of irreversible processes with internal variables [9] is to select relevant internal variables, for example z_i describing an internal working of the system and added as a complementary variable in the expression of any thermodynamic potential $\mathcal{P}$. For such an internal variable z_j, we also define π_j the *dual* thermodynamic force as $\pi_j = \frac{\partial \mathcal{P}}{\partial z_j}$. These additional variables permit to *explain* and to *compute* the Clausius-Duhem inequality and then to determine the dissipation resulting from the irreversibility of the processus:

$$đ\mathcal{D} = T \cdot đS_i = \sum_j \pi_j \cdot z_j \tag{13}$$

That permits to define a dissipation potential or a Rayleigh function for the lagrangian or hamiltonian formalism applied to a dissipative system.

III. Application of an MSMA based actuator

A. Characteristic of MSMAs and general description of the system

MSMAs are relatively new smart materials combining the properties of classical Shape Memory Alloys and the properties of magnetostrictive materials [10]. The thermo-magneto-mechanical behavior of such a material is quite difficult to model because of a strong nonlinear and hysteretical behavior [11] (c.f. FIG. 1 for an example of stress versus strain according to the magnetic field level).

For such a thermo-magneto-mechanical behavior, it is impossible to use quadratic Rayleigh function to describe the dissipation potential of the material and it makes sense to use the thermodynamics of irreversible processes with internal variables coupled with the lagrangian/hamiltonian formalism to propose a dynamical model of the system.

A scheme and a photograph of the MSMA device used as an exemple of the modeling procedure is presented on the FIG. 2. A magnetic circuit including a coil and a ferromagnetic core

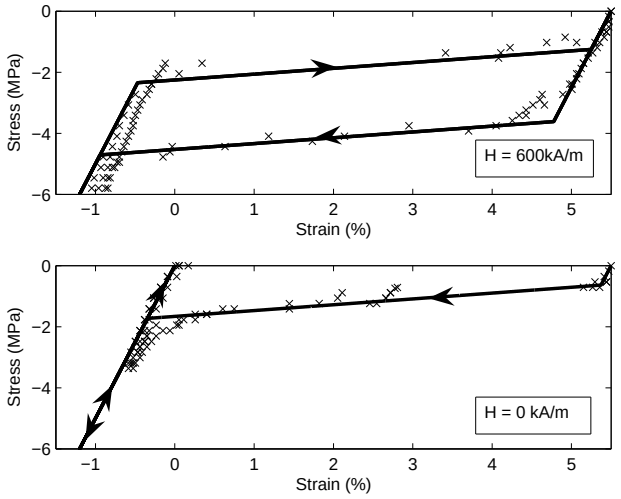


Fig. 1. Mechanical stress versus strain with and without a magnetic field for a cyclic charging (simulation: solid line, experimental results: cross points)

permits to create a magnetic field inside an air-gap where an MSMA sample is inserted. This sample is attached at one extremity to the fixed support and at the other extremity to a mobile load. The weight of the load permits to pre-stress the MSMA sample to obtain a motion in both directions. Gravitational and inertial effects of the load have to be taken into account. The coil is supplied by an home-made switching power amplifier (200 V - 2 A). The displacement of the load is measured with a laser sensor (Keyence LK-152) and the control is performed using a DSP board (dSpace). A PC is used for the displacement signal acquisition and to control the complete system.

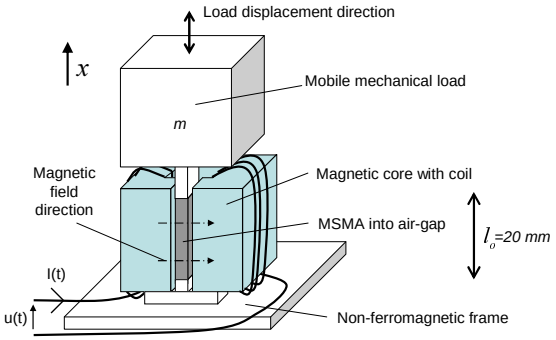
B. Modeling of the device

The modeling procedure is based on the computation of the Hamiltonian function for the full device. All the energetical terms of this function are expressed for each sub-system of the device:

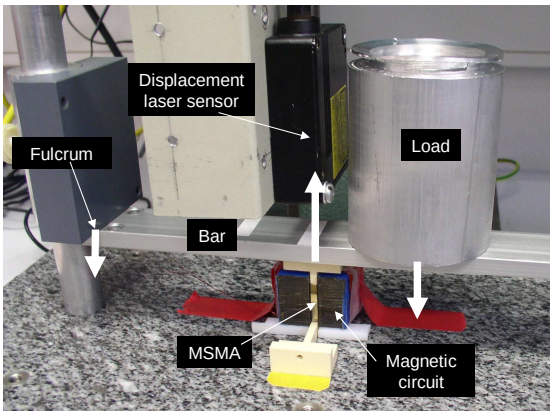
- the non-linear electromagnetical behaviour of the electrical (coil) and magnetical circuits (based on a lumped parameters circuit),
- the thermo-magneto-mechanical behaviour of the MSM material (based on the thermodynamics of irreversible processes),
- the mechanical behaviour of the load driven by the actuator (based on classical Lagrangian mechanics).

The different energetical terms of the full device are described on the FIG. 3. More details about each energetical expression may be found in [12].

1) Magnetic circuit: To model the core and the magnetic circuit, an electrical network including resistances and inductances was used. This lumped-parameters model permits to take into account the magnetic leakage in the surrounding air and the ferromagnetic saturation of the core without any time-consuming numerical computation such as the finite elements method. The different inductances in this electrical network are



(a)



(b)

Fig. 2. Description of the simple MSMA actuator: (a) scheme of the device, (b) photograph of the device.

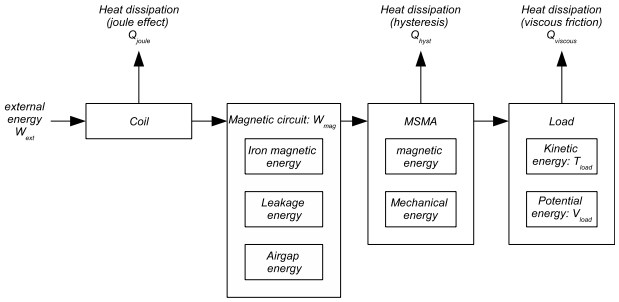


Fig. 3. Description of the different energies of the full device.

respectively associated with the magnetic flux accross the Fe-Si core, the leakage magnetic flux in the surrounding air, the magnetic flux accross the airgap, and lastly the magnetic flux accross the MSMA. It must be stressed that the inductances of the core and MSMA are not constant parameters but they take into account the nonlinear magnetic behaviours of these two materials. For the coil's quantities, a *global form* is used: the electric charge q_c is the generalized coordinate q_1, the current I is the generalized velocity $\dot{q}_1$, the magnetic flux ϕ is the generalized momentum p_1. For the other magnetic quantities, a *local form* is used: the magnetic excitation field D_i corresponds to the generalized coordinate, the curvilinear integral $H_i \cdot l_i$ along the path l_i corresponds to the generalized velocity, the flux $B_i \cdot S_i$ accross the surface S_i is the generalized momentum. l_{Fe}, l_l, l_a and l are respectively the ferromagnetic core, leakage, airgap and MSMA mean flux path lengths. The dissipation due to eddy-currents are neglected because the laminated Fe-Si magnetic core limits them drastically. Joule effect losses in the coil are taken into account with a *classical quadratic* dissipation potential (Rayleigh function $\mathcal{R}_1$).

The magnetic energy $\mathcal{W}_{mag}$ in this circuit depends on the magnetic fields B_i (generalized momentum p_i) in the volume V and we have:

$$\mathcal{H} = \mathcal{W}_{mag} = \int_V \int_o^{B_i} H_i(b) db \cdot dV \tag{14}$$

The magnetic energy stored in the Fe-Si core takes into account the nonlinear saturated magnetic behaviour of the Fe-Si material with an `arctan` shape function. The generalized external force applied to this sub-system is the voltage applied to the coil $f_{ext} = u(t)$. Ampère's law (Kirchhoff's Current Law) gives some algebraic relations between coordinates resulting in the definition of two kinematic constraints $c_1(\mathbf{q}) = 0$ and $c_2(\mathbf{q}) = 0$ (c.f. [12]).

2) MSMA: Two types of generalized coordinates are considered in the MSMA modelling. The first ones, the temperature T, the strain ε and the magnetic field H are *classical* thermodynamic variables. They are associated with three thermodynamic forces, the entropy s, the mechanical stress σ and the magnetization of the MSMA M. The coordinate T is not used afterwards because of the isothermic working of the actuator and the constant value of thermic part of the Helmholtz free energy $\mathcal{F}_{therm}$. The second type of generalized coordinates appears only in the frame of the thermodynamics of irreversible processes with internal variables. As explained previously, an internal variable is a generalized coordinate characterizing an *internal working* of the material not directly linked to any external forces. This variable permits to take into account the memory effect of the material. For the MSMA, the volume fraction z is such an internal variable (c.f. [11]).

The Hamiltonian function of the MSMA sample corresponds to its total energy. Because of the size and weight of the MSMA sample compared to the size and weight of the load, the influence of the potential and the kinetic energies of MSMA is quite low in the complete device energy and therefore these two terms can be neglected. The hamiltonian function of the MSMA sample of volume V_{MSMA} can be expressed as:

$$\mathcal{H}_{MSMA} = V_{MSMA} \cdot (\mathcal{F}_{mech} + \mathcal{F}_{mag}) + \frac{p_z^2}{2m_z} + \frac{p_\varepsilon^2}{2m_\varepsilon} \tag{15}$$

with $\mathcal{F}_{mech}$ and $\mathcal{F}_{mag}$, the mechanical and magnetical Helmholtz free energies, m_z and m_ε inertial parameters corresponding respectively to the z and ε variables.

The internal variable z was introduced to model the dissipative hysteretical behaviour of the material. In order to

satisfy the second thermodynamic law, z is used to define the Clausius-Duhem inequality:

$$đ\mathcal{D} = \pi^{f*}(z, \dot{z}) \cdot đz \geq 0 \tag{16}$$

with $\pi^{f*}(z, \dot{z})$ the thermodynamic force associated with z. The expression of $\pi^{f*}(z, \dot{z})$ will not be detailled in this paper and can be found in [12] but this expression can be used to express the dissipation power $\mathcal{P}_{hyst}$.

3) Load: The driven load energy includes a kinetic energy ($\mathcal{T}_{load} = \frac{1}{2m} p_x^2$ with m the mass of the load) and a gravity potential energy ($\mathcal{V}_{load} = mgx$ with g the gravity constant). The viscous friction of the load in the surrrounding air is modelled using a quadratic dissipation potential (Rayleigh function $\mathcal{R}_2 = \frac{f}{2}\dot{x}^2$ with f the viscous friction coefficient).

Moreover the load attached to the MSMA sample gives an algebraic relation between the strain ε and the displacement x and gives an other kinematic constraint $c_3(\mathbf{q})$.

By adding all the previous sub-system energies, the Hamiltonian function $\mathcal{H}$ of the full system can be written and this expression permits to obtain 16 Hamilton equations. Among these 16 equations, 8 are associated with the time rate of coordinates and 8 with the time rate of momentums. The 8 equations associated with the time rate of coordinates gives 8 definitions of variables:

- the first Hamilton equation is the definition of the inductance creating the magnetic flux ϕ,
- the four following Hamilton equations are the definition of the magnetic fields H_i,
- the three following Hamilton equations are the definition of the relations between the momentum p_i and the velocities $\dot{q}_i$ for $q_i \in \{z, \varepsilon, x\}$.

The 8 equations associated with the time rate of momentums give 8 relations that can be rewritten to obtain 4 physical equations, one constitutive equation for the MSMA and finally the value of the three Lagrange multipliers:

- the dynamic electrical equation (Voltage Kirchoff's Law),
- two equations for the conservation of magnetic fluxes in the magnetic circuit,
- the dynamic equation of the load (Newton's law),
- the quasi-static behaviour of the MSM material (the constitutive equation),
- the values of the three Lagrange multipliers

C. Discussion on the quasi-static and dynamic behaviour and on the energy distribution

To verify the prediction of the model, a sufficiently slow voltage ramp was applied as an input to verify the quasi-static model behaviour. The Fig. 4 presents the voltage, current and displacement of the actuator versus time for the experimental and the simulation values. The maximum reachable displacement is about 550 micrometres after a current ramp satured at 1 A. Then, a voltage step is applied to extract the dynamical behaviour of this system. The Fig. 5 reports also the voltage, current and displacement versus time (this graph uses a smaller time range). The maximum reachable displacement is now about 750 micrometres for a 1 A current step.

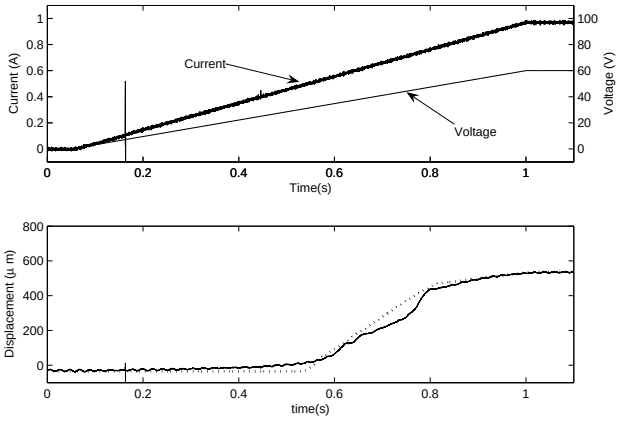


Fig. 4. Quasi-static behaviour of the system: voltage, current and displacement versus time (simulation: dotted line, experimental results: solid line).

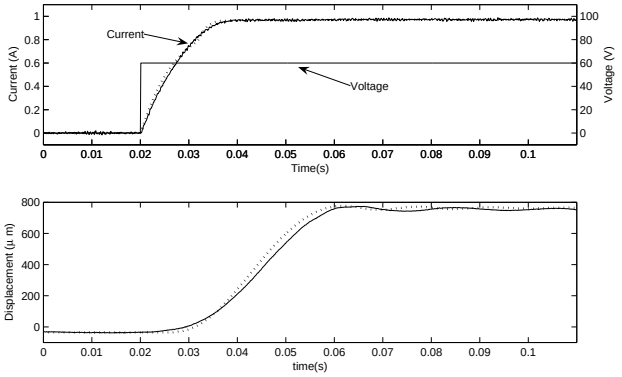


Fig. 5. Dynamic behaviour of the system: voltage, current and displacement versus time (simulation: dotted line, experimental results: solid line).

It is interesting to mention that the dynamical effect combined with the nonlinear MSMA behaviour permits to obtain a larger reachable strain in the dynamic mode than in the quasi-static mode. This is due to the decrease of the compressive stress applied to the MSMA sample when the acceleration of the load appears in dynamic mode. The simulation results appear reasonably accurate compared with the experimental measurements. The computation of different energetical terms of this MSMA simple actuator were computed.

The external energy $\mathcal{W}_{ext}$ supplied to the device is an electrical ones. As the energy exchange is $d\mathcal{W}_{ext} = u(t) \cdot dq_c$, the energy supply rate corresponding to the the electrical power is therefore $\dot{\mathcal{W}}_{ext} = u(t) \cdot I(t)$. The heat exchange was not measured on the experimental bench but the dissipation by Joule effect can be computed as $\mathcal{Q}_{joule} = \int_0^t \mathcal{R}_1(\dot{q}_1)\, dt$. The computations show that the main part of the supply energy is dissipated as heat losses into the coil. This confirms that MSMA as well as *classical* Shape Memory Alloys are not attractive materials from the efficiency point of view.

The rest of the available energy is divided into the coil and core magnetic energies (recoverable energies) and into

an energy transfer to the MSMA and to the load. A part of the MSMA energy is lost in the hysteretical loop of the material behavior $\mathcal{Q}_{hyst} = \int_0^t \mathcal{P}_{hyst}\, dt$ when the other is converted through the electromechanical energy conversion process. The result of this energy conversion is then distributed as a viscous friction process $\mathcal{Q}_{viscous} = \int_0^t \mathcal{R}_2(\dot{x})\, dt$, a potential energy $\mathcal{V}_{load}(x)$ and a kinetic energy $\mathcal{T}_{load}(p_x)$. We observe that the practical available mechanical energies (kinetic $\mathcal{T}_{load}$ and potential $\mathcal{V}_{load}$) are quite small compared to the input energy. In the quasi-static mode, the MSMA elastic energy corresponding to the mechanical Helmholtz free energy $\mathcal{F}_{mech}$ minus its interaction part $\mathcal{F}_{int}$ (due to the compatibility between martensite variant, c.f. [12]), the kinetic energy $\mathcal{T}_{load}$, and viscous losses $\mathcal{Q}_{viscous}$ are constant.

The differences between dynamic mode and quasi-static mode can also be discussed. In the dynamic working mode, an energy transfer between the MSMA elastic energy and the kinetic energy $\mathcal{T}_{load}$ exists. Actually, this elastic energy increases when some energy is supplied to the system, then decreases to a lower value than beginning because of kinetic energy $\mathcal{T}_{load}$. At this time, $\mathcal{V}_{load}$ can increase to a higher value than in quasi-static mode. The dissipation is more important for $\mathcal{Q}_{hyst}$ and $\mathcal{Q}_{viscous}$ in the dynamic case than in the quasi-static but $\mathcal{Q}_{joule}$ is lower in the dynamic case because the time range is smaller than in quasi-static mode.

It should be stressed that for this actuator, less than 8 to 11 mJ are recovered by the load for 5 to 20 Joules supplied to the actuator. These results clearly show a poor actuator efficiency. Because the main energetic losses are due to the Joule effect, an efficient actuator is an actuator which can hold a displacement value without keeping a current into the coil. This problem can be partially solved by using a Push-Pull actuator design working by voltage pulses: two MSMA samples and two magnetic circuits are used in an antagonistic way in order to obtain a multi-stable actuator. This kind of actuator was also designed and studied by the authors in [13], [14].

IV. Conclusion

The purpose of this paper was to present on a simple MSMA actuator example that the variational and energetical methods is not only well-suited to model *classical* micro-mechatronic devices such as these described on references [6] [7] but that they are also well-suited to include complex dynamical behaviour such as non-linearity and hysteretical behaviour. On this paper, the choice was made to applied these modeling techniques to a simple MSMA based actuator, nevertheless it could also be conveniently applied to any other micro-mechatronic systems using other non-linear and hysteretical smart materials. In a next paper, it will also be shown that this variational and energetic modeling procedure can be easily extented up to the control level because some important tools based on energetical concepts exist in the control field to explore the stability and to design efficient control laws (Lyapunov function, dissipativity, passivity, energy shaping, port-hamiltonian modeling as reported in [15], [16] and [17]).

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